THE PRINCIPLE OF LEAST WORK IN MECHANICS AND ITS USE IN INVESTIGATIONS REGARDING THE ETHER OF SPACE.

BY MANSFIELD MERRIMAN.

(*Read April 2, 1903.*)

The principle of least work has been extensively used in applied mechanics since 1879, when it was first formally stated and established by Castigliano. Previous to that time, various authors had discussed the principles of least action, of least constraint, and of least resistance, and had applied them in the solution of special problems. The principle of least work, however, is capable of more definite statement and demonstration than the other minimum laws, and its range of application in statical investigations on elastic structures is wide, while it has been found to be of great practical value to civil engineers.

When a structure like a bridge truss contains members sufficient to prevent distortion of its panels and no more, the stresses in these members due to given loads can be readily computed by the principles of rigid statics, the members in this case being called necessary ones. If there be superfluous members, however, rigid statics cannot determine the stresses, since the number of unknown stresses is greater than the number of statical conditions. In this case the structure is said to be statically indeterminate, and the principle of least work must be applied. This principle asserts that the stresses under consideration have such values that the potential stress energy stored in all the members of the structure shall be a minimum. If there be n stresses under consideration and m statical conditions, the remaining $n - m$ conditions are expressed by $n - m$ equations, which are deduced by equating to zero the derivatives of the expression for the total stored energy, these being the conditions that render this energy a minimum.

As a simple example the case of a rectangular table with four legs may be considered, it being required to find the stresses in these legs due to single load placed on the table in a given position. This is a statically indeterminate problem, since rigid statics furnishes but three conditions, and the solution cannot be made if the legs are rigid. The legs are, however, really elastic and each one is shortened in supporting the load, the stress in each leg multiplied by the amount of shortening being proportional to the stored energy in it. The amount of shortening is, moreover, proportional to the stress, if the elastic limit of the material be not exceeded. Accordingly, the stress energy in the four legs due to the given load is proportional to the sum of the squares of the four stresses, and this sum is to be made a minimum. This condition, in connection with the three statical ones, enables the four stresses due to the load to be readily determined for any given position of that load, and that these stresses actually occur is easily verified by experiment.

A close analysis of the principle of least work as applied to any framed structure will show that its applicability and its validity depend upon the fact that the longitudinal deformation of any member is proportional to the stress upon it. This law of elasticity, commonly known as Hooke's law, is closely true for the materials used in engineering structures, provided the elastic limit be not exceeded. In all cases of the design of structures it is intended that this limit shall not be surpassed, and hence the principle of least work may be used with confidence and success in computations of stresses in statically indeterminate trusses.

It is sometimes asserted that the principle of least work is a statement of a general law of nature which is obeyed not only by materials under stress but by animate beings. While it may be true that men and animals endeavor to perform their tasks in the way most economical of effort, this analogy has no bearing upon the demonstration of the principle of least work. For this demonstration rests upon the theorem of virtual velocities, the formula for the stored stress energy being the integral of that of virtual velocities. On analyzing this proof it is seen that the integration is rendered possible by the fact that the deformation of each member is assumed to be proportional to the stress upon it. This assumption indeed is the same as that of the superposition of forces, for it supposes each stress to produce its effects independently of the existence of other stresses. The theorem of virtual velocities applies to all cases of equilibrium, but its integral form does not give the principle of least work unless Hooke's law of elasticity is fulfilled. This principle, therefore, is of limited application in mechanics, and it states no general law of nature.

In the method of least squares the conditions and rules for finding the most probable values of observed quantities are derived from the principle that the sum of the squares of the residual errors shall be a minimum. In theoretical mechanics the condition for finding the centre of mass of a system of bodies may be expressed by saying that the moment of inertia of the system shall be a minimum. In the mechanics of elastic bodies the principle of least work is analogous to these, for the conditions which must be fulfilled are those found by making the stored energy of the system a minimum. In all these cases the algebraic conditions are expressed by linear equations while the laws from which they are derived are in quadratic form, and these laws are only true when each elementary error or particle produces its effects independently of others.

Solid beams and tubes, as well as framed trusses, are subject to the principle of least work, provided the materials of which they are made conforms to Hooke's law of elasticity. For instance, the thick hollow cylinder of a gun tube is stressed under the pressure arising from the explosion of the powder, and the stress at any point varies inversely as the square of the distance between that point and the centre of the tube. It is easy to show that this law of variation is the one which makes the stored energy in the tube a minimum. So in a hollow sphere subject to interior pressure, the stresses throughout the spherical annulas vary inversely as the cubes of their distances from the centre, and this law of variation is the one which renders the stored stress energy a minimum.

The ether that fills space and transmits the force of gravitation from every particle of matter to all others has been regarded by many physicists as an elastic solid which obeys Hooke's law. If so it must be subject to the principle of least work. Any portion of matter may be supposed to exert upon the ether a compressive force, due to the fact that its molecules have displaced the ether and crowded it outwards. Then the stresses in the ether due to this displacement must be so distributed that the stored energy in the infinite sphere may be a minimum. Stating the algebraic expression for this energy due to a spherical body, it is found that its minimum value occurs when the stress at any point in the ether varies inversely as the cube of the distance of that point to the centre of the body. If gravitation be a differential effect, due to the difference of the stresses upon opposite sides of a body, the force of attraction between two spheres should vary inversely as the fourth power of the distance between their centres. From no point of view does it seem possible to deduce the actual law of gravitation from the stresses which must exist in the ether under the supposition of perfect elasticity.

The use of the principle of least work in investigations regarding the ether of space hence leads to negative results, as far as its applicability is concerned. It indicates, however, the important conclusion that the ether is not an elastic substance in which stresses are proportional to deformations, and accordingly studies concerning it should be based upon other suppositions concerning its properties. Since the ether cannot be made the object of experiment and since all we know concerning it is from rough analogy and indirect evidence, negative conclusions are valuable. By successively discussing and rejecting one assumption after another, it is possible that in due time properties of the ether may be found which will explain not only the inertia and gravitation of matter, but also the phenomena of electricity and magnetism.

LEHIGH UNIVERSITY, SOUTH BETHLEHEM, PA.

THE "FRANKLIN PAPERS" IN THE AMERICAN PHILOSOPHICAL SOCIETY.

BY J. G. ROSENGARTEN.

(*Read April 3, 1903.*)

In the collection of this Society there are some seventy large folio volumes of "Franklin Papers." Franklin left all his papers to his grandson, William Temple Franklin, who, after a long interval, published in London and Philadelphia six volumes of Franklin's works. Of course, this represented but a small part of his papers. Those used in the preparation of Temple Franklin's edition are now the property of the United States, which has never yet printed a Calendar of them. Temple Franklin selected from his grandfather's papers those that he thought suitable for publication, and left the rest in charge of his friend, Charles Fox, to whom he bequeathed them, and Charles Fox's heirs, in turn, after a long lapse of years, presented them to the American Philosophical Society, in whose custody they have remained ever since. They have been roughly classified, and are bound in a rude and careless way. Under the present efficient Librarian, Dr. Hays, a Calendar is being made as fast as the limited means at his disposal will per